

M-ary PULSE AMPLITUDE MODULATION

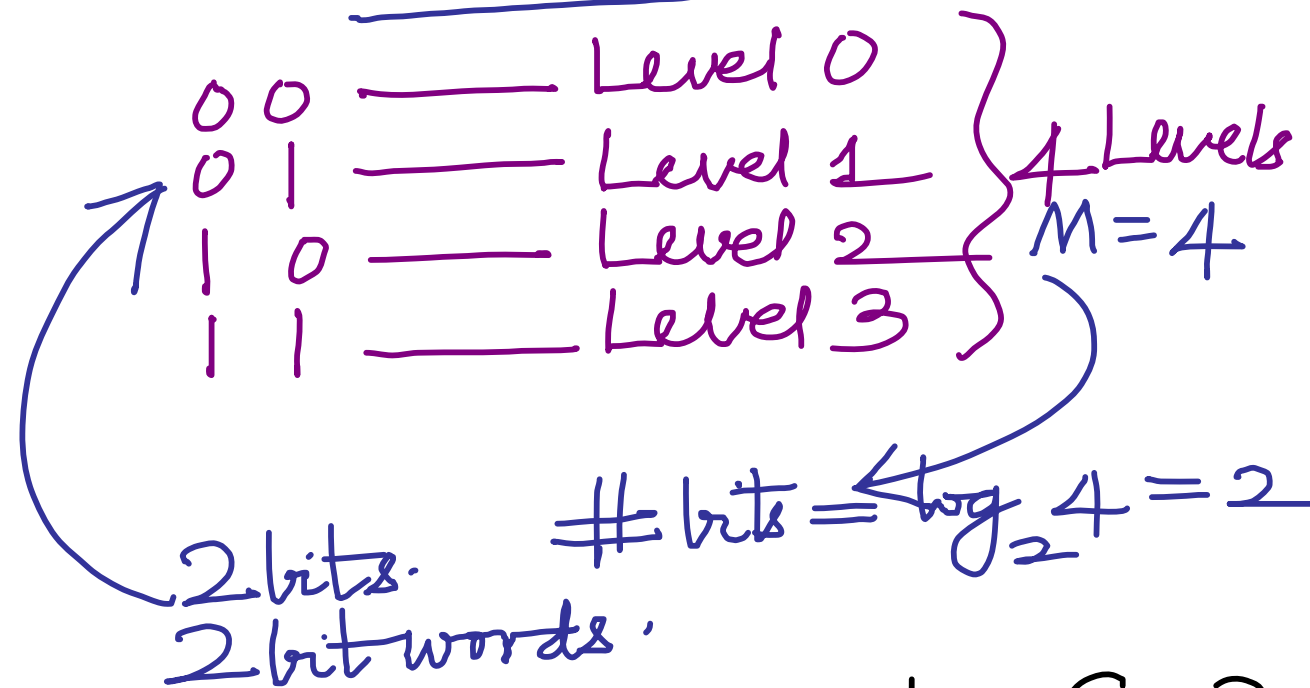
M-Levels.
M-ary (PAM)

waveform

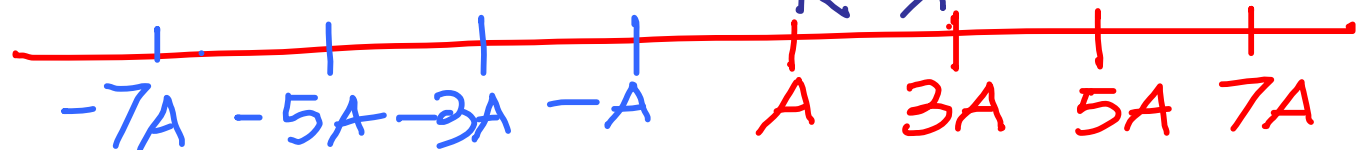
$$x(t) = a_p(t)$$

can take one of
M different Levels.

$$\# \text{ bits} = \log_2 M \text{ bits per symbol.}$$



Ex: $M = 8 \Rightarrow \# \text{ bits} = \log_2 8 = 3$
 8 possible levels.
 Distance between levels = $2A$



$$a_0 \in \{\pm A, \pm 3A, \pm 5A, \pm 7A\}$$

General M-ary PAM:

$$a_0 = \pm (2i+1)A$$

$$i = 0, 1, 2, \dots, \frac{M}{2} - 1$$

M is even

$$\boxed{\# \text{ bits/symbol} = \log_2 M}$$

Let average symbol Energy
 $= E_s$.

$$E_s = \frac{1}{M/2} \sum_{i=0}^{\frac{M}{2}-1} (2i+1)^2 A^2$$

$$a_0 = (2i+1)A$$

By Symmetry

$$= \frac{2A^2}{M} \sum_{i=0}^{\frac{M}{2}-1} (4i^2 + 4i + 1)$$

Standard
Expressions
 $\sum_{i=0}^{\frac{M}{2}-1} i^2$

$$= \frac{2A^2}{M} \left\{ \frac{4 \cdot \left(\frac{M}{2}-1\right) \frac{M}{2} (M-1)}{6} + \frac{4 \left(\frac{M}{2}-1\right) \frac{M}{2}}{2} + \frac{M}{2} \right\}$$

$$= \frac{2A^2}{M} \left\{ \frac{2}{3} \cdot \left(\frac{M^2}{4} - \frac{M}{2} \right) (M-1) \right. \\ \left. + M \left(\frac{M}{2} - 1 \right) + \frac{M}{2} \right\}$$

$$= \frac{2A^2}{M} \cdot \left\{ \frac{2}{3} \left(\frac{M^3}{4} - \frac{3M^2}{4} + \frac{M}{2} \right) \right. \\ \left. + \frac{M^2}{2} - \frac{M}{2} \right\}$$

$$= \frac{2A^2}{M} \cdot \left\{ \frac{M^3}{6} - \frac{M}{6} \right\}$$

$$= \frac{2A^2}{M} \cdot \frac{M}{6} \cdot (M^2 - 1)$$

$$= \frac{A^2}{3} \cdot (M^2 - 1)$$

Function of A, M

$$E_s = \frac{A^2}{3} \cdot (M^2 - 1)$$

Average symbol
Energy

Spacing between M -ary
PAM Levels $= 2A$!

$$E_s = \frac{A^2}{3} \cdot (M^2 - 1).$$

$$A = \sqrt{\frac{3E_s}{M^2 - 1}}$$

Ensures average symbol Energy of E_s .

$$x(t) = a_i p(t).$$

$$\pm (2i+1)A$$

$0 \leq i \leq \frac{M}{2} - 1.$

Choose pulse $p(t)$.

$$p(t) = \sqrt{\frac{2}{T}} \cdot \cos(2\pi f_c t) \quad 0 \leq t \leq \frac{K}{f_c}.$$

Energy of Pulse
 $= E_p = 1$

Receiver for M-ary PAM

Matched Filter

$$h(t) = p(T-t).$$

After Matched Filtering & Sampling at $t = T$.

$$r(T) = a_0 E_p + \tilde{n}$$

$$= a_0 + \tilde{n}$$

$$a_0 = \pm (2i+1)A$$

Assuming input noise is Gaussian Zero-mean, with

$$R_{nn}(\tau) = \frac{N_0}{2} \delta(\tau)$$

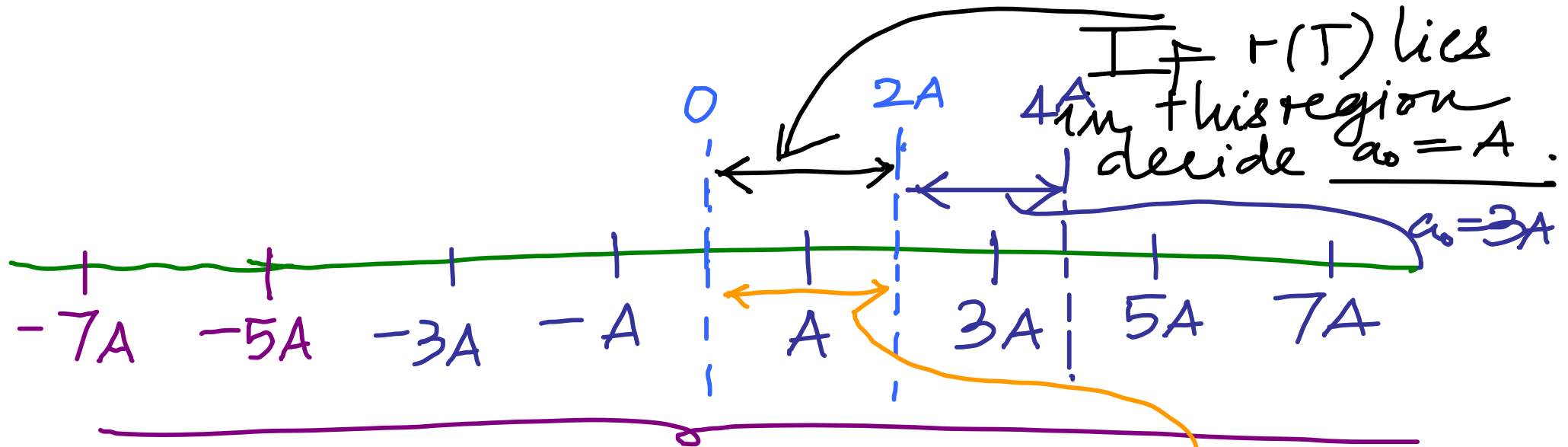
Gaussian Noise

Mean = 0

$$\text{var} = \frac{N_0}{2} E_p = \frac{N_0}{2}$$

Additive White Gaussian Noise.

Probability of Error:



$M = 8$ PAM constellation

Closest Neighbor = A.

By Symmetry:

I_T^0 $0 < I(T) < 2A$
 $\Rightarrow \text{Decision} = A$

Internal
Points

$$2A \leq r(T) < 4A$$

$$\Rightarrow \text{Decision} = 3A$$

$$-2A \leq r(T) < 0$$

$$\Rightarrow \text{Decision} = -A$$

⋮

Combination of
M-cases

← one for
each point.

Nearest Neighbor
Decision rule.

Assigning $r(T)$ to the
closest constellation pt.